Course: Numerical Analysis I (Elective A)
Course Code: USMT5A4/UAMT5A4

T.Y.B.Sc Paper iv

Numerical Analysis I(Elective A)				
USMT5A4 ,UAMT 5A4	I	Errors Analysis	2.5	3
	II	Transcendental and Polynomial & Equations		
	III	Linear System of Equations		

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N.B. Derivations and geometrical interpretation of all numerical methods have to be covered.

Unit I. Errors Analysis and Transcendental & Polynomial Equations (15L)

Measures of Errors: Relative, absolute and percentage errors. Types of errors: Inherent error, Round-off error and Truncation error. Taylors series example. Significant digits and numerical stability. Concept of simple and multiple roots. Iterative methods, error tolerance, use of intermediate value theorem. Iteration methods based on first degree equation: Newton-Raphson method, Secant method, Regula-Falsi method, Iteration Method. Condition of convergence and Rate of convergence of all above methods.

Unit II. Transcendental and Polynomial Equations (15L)

Iteration methods based on second degree equation: Muller method, Chebyshev method, Multipoint iteration method. Iterative methods for polynomial equations; Descarts rule of signs, Birge-Vieta method, Bairstrow method. Methods for multiple roots. Newton-Raphson method. System of non-linear equations by Newton- Raphson method. Methods for complex roots. Condition of convergence and Rate of convergence of all above methods.

Unit III. Linear System of Equations (15L)

Matrix representation of linear system of equations. Direct methods: Gauss elimination method.

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Matrix representation of linear system of equations. Direct methods: Gauss elimination method.

Pivot element, Partial and complete pivoting, Forward and backward substitution method, Triangularization methods-Doolittle and Crouts method, Choleskys method. Error analysis of direct methods. Iteration methods: Jacobi iteration method, Gauss-Siedal method. Convergence analysis of iterative method. Eigen value problem, Jacobis method for symmetric matrices Power method to determine largest eigenvalue and eigenvector.

Recommended Books

1. Kendall E. and Atkinson, An Introduction to Numerical Analysis, Wiley.

2. M. K. Jain, S. R. K. Iyengar and R. K. Jain, Numerical Methods for Scientific and Engineering Computation, New Age International Publications.

1. Duration: The examinations shall be of 3 Hours duration.

2. Theory Question Paper Pattern:

   a) There shall be FIVE questions. The first question Q1 shall be of objective type for 20 marks based on the entire syllabus. The next three questions Q2, Q2, Q3 shall be of 20 marks, each based on the units I, II, III respectively. The fifth question Q5 shall be of 20 marks based on the entire syllabus.

   b) All the questions shall be compulsory. The questions Q2, Q3, Q4, Q5 shall have internal choices within the questions. Including the choices, the marks for each question shall be 30-32.

   c) The questions Q2, Q3, Q4, Q5 may be subdivided into sub-questions as a, b, c, d & e, etc and the allocation of marks depends on the weightage of the topic.

   d) The question Q1 may be subdivided into 10 sub-questions of 2 marks each.

Question Paper pattern:

Paper pattern: The question paper shall have two parts A, B.
Each part shall have two Sections.

Section I Objective in nature: Attempt any Eight out of Twelve multiple choice questions. ($8 \times 3 = 24$ Marks)

Section II Problems: Attempt any Two out of Three. ($8 \times 2 = 16$ Marks)

Marks for Journals and Viva:
For each course USMT501/UAMT501, USMT502/UAMT502, USMT503/UAMT503, USMT504/UAMT504, USMT601/UAMT601, USMT602/UAMT602 USMT603/UAMT603, and USMT604/UAMT604:

1. Journals: 5 marks.

2. Viva: 5 marks.

Each Practical of every course of Semester V and VI shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the journal. A student must have a certified journal before appearing for the practical examination.

Error function

- True value – Approximate value = Error

$$Relative\ Error = \frac{|Error|}{|True\ value|} \qquad Absolute\ error = |Error|$$

- The **inherent** error is that quantity which is already present in the statement of the problem before its solution
- The **round-off error** is the quantity R which must be added to the finite representation of a computed number in order to make it the true representation of that number.
- The **truncation error** is the quantity T which must be added to the true representation of the quantity in order that the result be exactly equal to the quantity we are seeking to generate.

Newton-Raphson method

- Formula.

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$x_{k+1} = x_k - m\frac{f(x_k)}{f'(x_k)}$$

Multiple root of multiplicity m

Intermediate Value Theorem: If f(x) is a continuous function on some interval [a,b] and f(a) f(b)< 0, then the equation f(x) = 0 has at least one or an odd number of real roots in the interval (a,b).

Qn:Determine the initial approximation to find the smallest positive root. also find the smallest root correct to 4 decimal places

$$f(x) = x^4 - x - 10$$

Solution:

$f(1) = -10$ and $f(2) = 4$

The interval (1,2) is the smallest positive root lies.

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

Put $x_0 = 2$, we get

$x_1 = 1.8710$, $x_2 = 1.8558$, $x_3 = 1.8556$, $x_4 = 1.8556$

The smallest root is 1.8556

Qn:Determine the initial approximation to find the smallest positive root. also find the smallest root correct to 4 decimal places $f(x) = x - e^{-x}$

- Solution:

  f(0) =-1, f(1) = 0.6321. The smallest positive root is lying between (0,1)

  Put $x_0 = 1$, we get

  $x_1 = 0.5379$, $x_2 = 0.5670$, $x_3 = 0.5671$, $x_4 = 0.5671$

  The smallest root is 0.5671

Secant Method

In this we need 2 initial approximate values x_{k-1} and x_k . The order of this method is 1.62

$$x_{k+1} = \frac{x_{k-1} f_k - x_k f_{k-1}}{f_k - f_{k-1}}$$

$$\frac{A\,f(B) - B\,f(A)}{f(B) - f(A)}$$

If the approximations are chosen such that $f(x_k).f(x_{k-1}) < 0$ for each k, then the method is known as Regula-Falsi method and has the first order rate of convergence.

Qn:Determine the initial approximation to find the smallest positive root. also find the smallest rot correct to 4 decimal places $f(x) = x^4 - x - 10$

Solution:

$f(1) = -10$ and $f(2) = 4$

The interval (1,2) is the smallest positive root lies.

Put $x_0 = 1$ and $x_1 = 2$ we get

$x_2 = 1.7143$, $x_3 = 1.8385$, $x_4 = 1.8578$, $x_5 = 1.8556$, $x_6 = 1.8556$

The solution is 1.8556

Determine the initial approximation to find the smallest positive root. also find the smallest rot correct to 4 decimal places $f(x) = x - e^{-x}$

- Solution:

$f(0) = -1$, $f(1) = 0.6321$. The smallest positive root is lying between (0,1)

$f(x) = x - e^{-x}$, $x_0 = 0$, $x_1 = 1$

$x_2 = 0.6127$, $x_3 = 0.5638$, $x_4 = 0.5671$, $x_5 = 0.5671$

- Regula-falsi method

$x_2 = 0.6127$, $x_3 = 0.5722$, $x_4 = 0.5677$, $x_5 = 0.5672$

Chebyshev Method

- Write $f(x) = f(x_k + x - x_k)$ and approximating f(x) by a second drgree Tayler series expansion about the point x_k, we obtain

$$x_{k+1} = x_k - \frac{f_k}{f_k{}'} - \frac{1}{2}\left(\frac{f_k}{f_k{}'}\right)^2 \frac{f_k{}''}{f_k{}'} \qquad k = 0,1,2, \dots \dots$$

- Whose order is p=3. This method requires one function, one first derivative and one second erivative evaluation per itration.

Ex: Find the smallest root correct to 6 decimal places by Chebyshev Method $f(x) = x^3 - 5x + 1$, take $x_o = 0.5$

Solution:

$f'(x) = 3x^2-6$, $f''(x) = 6x$

$f(0.5) = -1.375$, $f'(0.5) = -4.25$, $f''(0.5) = 3$

$$x_{k+1} = x_k - \frac{f_k}{f_k{}'} - \frac{1}{2}\left(\frac{f_k}{f_k{}'}\right)^2 \frac{f_k{}''}{f_k{}'} \quad k = 0,1,2, \ldots \ldots$$

$x_1 = 0.213414$

$f(0.213414) = -0.057350$, $f'(0.213414) = -4.863363$,
$f''(0.213414) = 1.280484$

$x_2 = 0.201640$

Ex: Find the smallest root correct to 6 decimal places by Chebyshev Method

$$f(x) = \cos x - xe^{x}$$ take x_o= 1.0

Solution:

$$f'(x) = -\sin x - (x+1)e^{x} \quad f''(x) = -\cos x - (x+2)e^{x}$$

f(1) = -2.17797952, f'(1) = -6.27803464, f''(1) = -8.69514779

$$x_{k+1} = x_k - \frac{f_k}{f_k{'}} - \frac{1}{2}\left(\frac{f_k}{f_k{'}}\right)^2 \frac{f_k{''}}{f_k{'}} \qquad k = 0,1,2, \ldots \ldots$$

x_1= 0.56973365

$f(x_1)$ = -0.16512827, $f'(x_1)$ = -3.31437687, $f''(x_1)$ = -5.38480989

x_2 = 0.51789543

$f(x_2)$ = -0.00042006, $f'(x_2)$ = -3.04282712, $f''(x_2)$ = -5.09512887

x_3 = 0.51775736

Ex: Find the approximate value of 1/7 by Chebyshev Method, do two iteration take $x_o = 0.1$

Solution:

$$f(x) = \frac{1}{x} - 7 \qquad f'(x) = -\frac{1}{x^2} \qquad f''(x) = \frac{2}{x^3}$$

$x_0 = 0.1$

$f(x_0) = 3$, $f'(x_0) = -100$, $f''(x_0) = 2000$, substitute in the following

$$x_{k+1} = x_k - \frac{f_k}{f_k'} - \frac{1}{2}\left(\frac{f_k}{f_k'}\right)^2 \frac{f_k''}{f_k'} \qquad k = 0,1,2,\ldots\ldots$$

$x_1 = 0.1 + 0.03 - 0.5(0.0009)(-20) = 0.139$

$x_1 = 0.139$

$f(x_1) = 0.194245$, $f'(x_1) = -51.757155$, $f''(x_1) = 744.707272$

$x_2 = 0.139 + 0.003753 - 0.5\,(0.000014)(-14.388489) = 0.142854$

Multipoint iteration Method

$$x_{k+1} = x_k - \frac{f_k}{f'(x_k - \frac{f_k}{2f_k'})}, k = 0,1,2, \dots \dots$$

- This method need only one approximation and one derivative.

Ex: Find the smallest root correct to 6 decimal places by Multipoint Method
$f(x) = x^3 - 5x + 1$, take $x_o = 0.5$ do two iteration

Solution:
$f(x) = x^3 - 5x + 1, f'(x) = 3x^2 - 6$,

$x_o = 0.5$

$f(0.5) = -1.375$, $f'(0.5) = -4.25$, substitute in the formula

$$x_{k+1} = x_k - \frac{f_k}{f'(x_k - \frac{f_k}{2f_k'})}, k = 0,1,2, \dots \dots$$

$x_1 = 0.205445$

$f(x_1) = -0.018554$, $f'(x_1) = -4.873377$, substitute in the formula

$x_2 = 0.201640$

Ex: Find the smallest root correct to 6 decimal places by Multipoint Method

$f(x) = \cos x - xe^{x}$, take x_o= 1.0, three iteration

Solution:

$f(x) = \cos x - xe^{x}$ $f'(x) = -\sin x - (x+1)e^{x}$

, take x_o= 1.0,

f(1) = -2.17797952, f'(1) = -6.27803464, substitute in the formula

x_1= 0.57970578

$f(x_1)$ = -0.19844837, $f'(x_1)$ = -3.36836305, substitute in the formula

x_2 = 0.51804416

$f(x_2)$ = -0.00087268, $f'(x_2)$ = -3.04358498, substitute in the formula

x_3 = 0.51775736

Muller Method

- This need three initial values.

$$x_{k+1} = x_k - \frac{2a_2}{a_1 \pm \sqrt{(a_1^2 - 4a_0a_2)}}, k = 0,1,2, \dots \dots$$

- The sign of the denominator is chosen as that of a_1, so that the denominator has the maximum absolute value.

$$a_2 = f_k$$

$$a_1 = \frac{1}{D}\,[(x_k - x_{k-2})^2(f_k - f_{k-1}) - (x_k - x_{k-1})^2(f_k - f_{k-2})$$

$$a_2 = \frac{1}{D}\,[(x_k - x_{k-2}) \quad (f_k - f_{k-1}) - (x_k - x_{k-1}) \ (f_k - f_{k-2})$$

$$D = (x_k - x_{k-1})(x_k - x_{k-2})(x_{k-1} - x_{k-2})$$

- The rate of convergence of Muller method is 1.84

Ex: Find the smallest root by Muller Method for $f(x) = x^3 - 5x + 1$, (0,1) do three iteration

Solution:

$$x_0 = 0, x_1 = 0.5, x_2 = 1 \qquad f_0 = 1, f_1 = -1.375, f_2 = -3$$

$a_2 = -3$, $D = 0.25$, $a_1 = -2.5$, $a_0 = 1.5$

$$x_{k+1} = x_k - \frac{2a_2}{a_1 \pm \sqrt{(a_1^2 - 4a_0a_2)}}, k = 0,1,2, \ldots \ldots$$

$x_3 = 0.191857$, the sign of the denominator is negative since a_1 is negative.

$a_2 = -0.047777$, $D = 0.124512$, $a_1 = -5.138588$, $a_0 = 1.691854$

$x_4 = 0.201183$, the sign of the denominator is negative since a_1 is negative.

$a_2 = 0.002228$, $D = 0.006020$, $a_1 = -4.871483$, $a_0 = 1.393112$

$x_5 = 0.201640$, the sign of the denominator is negative since a_1 is negative.

Ex: Find the smallest root by Muller Method (0,1) do two iteration

$$f(x) = \cos x - xe^x$$

Solution:

$$x_0 = -1, x_1 = 0, x_2 = 1$$ f_0 =0.9082, f_1= 1, f_2 = -2.1780

a_2 = -2.1780, D = 2, a_1 = -4.8129, a_0 = -1.6349

$$x_{k+1} = x_k - \frac{2a_2}{a_1 \pm \sqrt{(a_1^2 - 4a_0a_2)}}, k = 0,1,2, \dots \dots$$

x_3 = 0.4415, the sign of the denominator is negative since a_1 is negative

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$$x_1 = 0, x_2 = 1, x_3 = 0.4415$$

f_1 =1, f_2= -2.178, f_3 = 0.2176

a_2 = 0.2176, D = 02466, a_1 = -2.8830, a_0 = -2.5170

x_4 = 0.5126, the sign of the denominator is negative since a_1 is negative.